Basic Geometric Ideas

A **point** is an exact location.	•
A **line** is a straight path of points that goes on forever in two directions.	←————————————→
A **ray** is a part of a line. A ray has one endpoint and goes on forever in one direction.	•————————————→
A line **segment** is a part of a line with two endpoints.	•————————————•
Congruent line segments are line segments that have the same length.	
The **midpoint** of a line segment is halfway between the endpoints of a line segment.	•————•————•
A **plane** is a flat surface that extends forever in all directions.	

Intersecting lines meet at exactly one point.	Parallel lines never meet. They are always the same distance apart.	Perpendicular lines form a 90° angle.

Use the diagram at the right. Name the following.
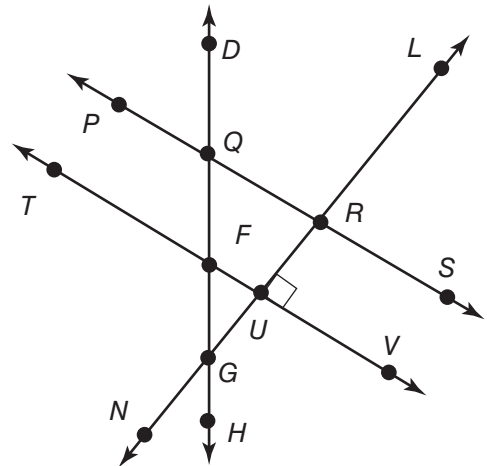
1. Three line segments
   Sample answers:
   $\overline{QF}$, $\overline{FH}$, $\overline{NG}$, $\overline{UR}$, $\overline{QS}$

2. Two parallel lines

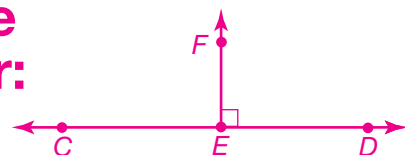   **Sample answers:** $\overleftrightarrow{PS}$, $\overleftrightarrow{TV}$

3. Two lines that intersect $\overleftrightarrow{PS}$

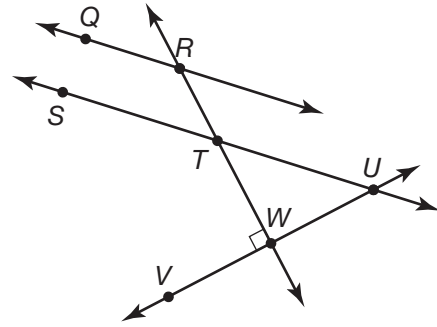   **Sample answers:** $\overleftrightarrow{DH}$, $\overleftrightarrow{NL}$

4. **Draw a Diagram** Draw a diagram in which the midpoint of $\overline{CD}$ is also the endpoint of $\overrightarrow{EF}$, which is perpendicular to $\overline{CD}$.

   Sample answer:

Basic Geometric Ideas

Use the diagram at the right. Name the following.

1. Two perpendicular lines $\overleftrightarrow{RW}$ **and** $\overrightarrow{VU}$

2. Two rays $\overrightarrow{QR}$ **and** $\overrightarrow{ST}$

3. Two parallel lines $\overleftrightarrow{QR}$ **and** $\overleftrightarrow{SU}$

4. Four line segments $\overline{ST}, \overline{RT}, \overline{TW}, \overline{WU}$
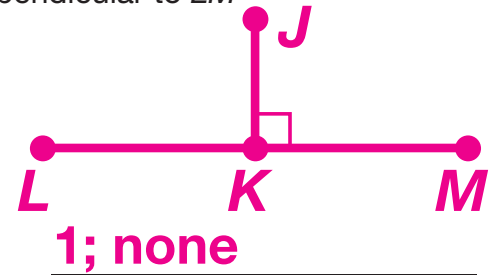
5. Two lines that intersect $\overleftrightarrow{SU}$ **and** $\overleftrightarrow{RW}$

Draw a diagram to illustrate each situation.
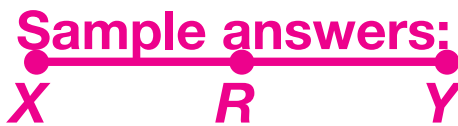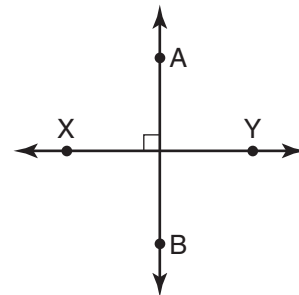
6. $\overline{XY}$ with midpoint R **7.** $\overline{JK}$ perpendicular to $\overline{LM}$

Sample answers:

X   **R**   Y

8. Reasoning How many points are shared by two perpendicular lines? By two parallel lines?

1; none

9. Which best describes the diagram?

    (**A**) Perpendicular lines

    B Parallel lines

    C Skew lines

    D. Intersecting lines

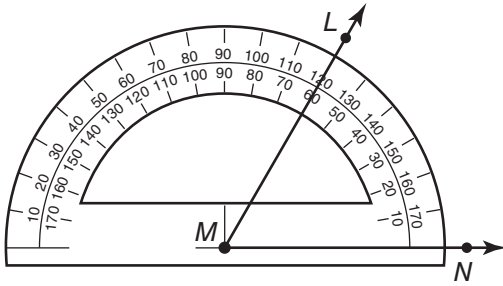**10. Writing to Explain** In your own words, describe a plane.

Sample answer: A plane is a flat surface and does not end.

Measuring and Drawing Angles

How to measure an angle:

Step 1 Place the protractor's center on the angle's vertex.

Step 2 Place the 0° mark on one side of the angle.

Step 3 Use the scale beginning with the 0° mark to read the measurement where the other side of the angle crosses the protractor.

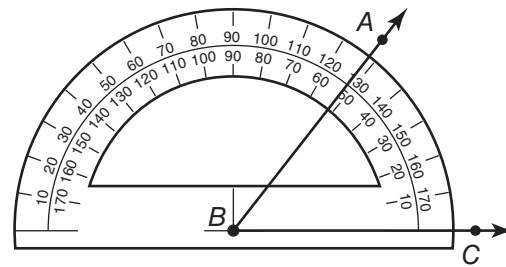m∠LMN = 60°

How to draw an angle:

Draw an angle of 52°.

Step 1 Draw a ray.

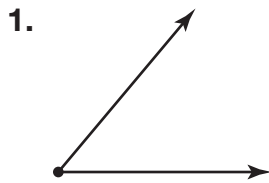**Step 2** Place the protractor's center on the endpoint. Line up the ray with the 0° mark.

Step 3 Using the scale with the 0° mark, place a point at 52°.
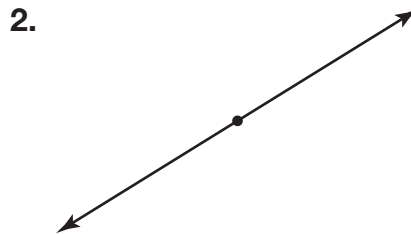
Step 4 Draw the other ray.

m∠ABC = 52°

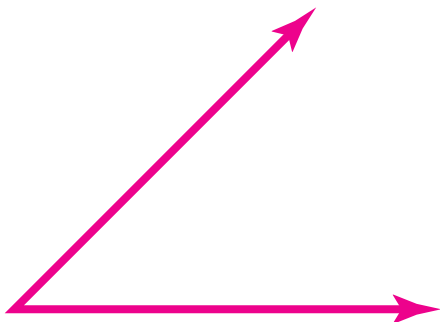Classify each angle as acute, right, obtuse, or straight. Then measure the angle.
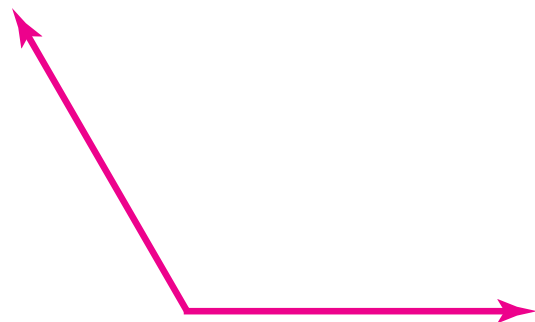
1.

Acute; 50°

2.

Straight; 180°
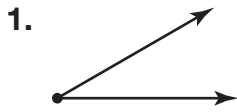
Draw an angle with each measure.

3. 45°

4. 120°

Measuring and Drawing Angles

Classify each angle as acute, right, obtuse, or straight. Then measure the angle.
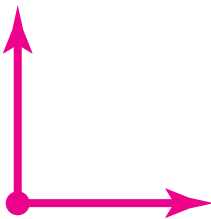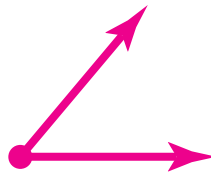
1.

Acute; 30°

2.

Obtuse; 150°

3.
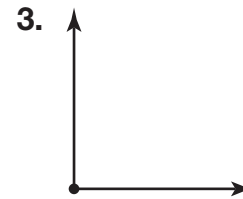
Right; 90°

Draw an angle for each measure.   **Sample answers for 4–9**

4. 90°

5. 50°

6. 112°
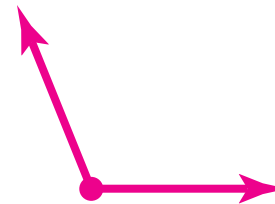
Estimation Without a protractor, try to sketch an angle with the given measure. Then use a protractor to check your estimate.

7. 120°
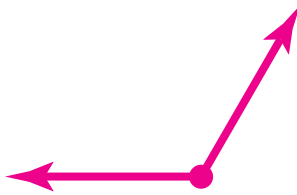
8. 100°
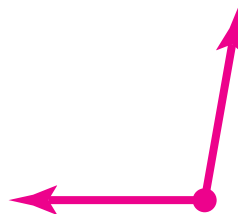
9. 10°

10. Which is a measure of an acute angle?

  (A) 40°   **B** 90°   **C** 120°   **D** 180°

11. Writing to Explain Explain the steps you use to measure an angle using a protractor.

Sample answer: Line up the protractor's center on an angle's vertex. Begin at the 0° mark and use the scale to measure.

Angle Pairs

Vertical angles are pairs of congruent angles created when two lines intersect.

150°
30° 30°
150°

Complementary angles are two angles that together can form a right angle.

The sum of their measures is 90°.

70°
20°

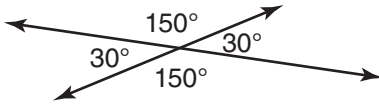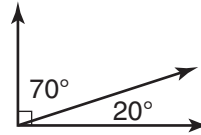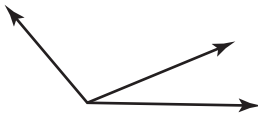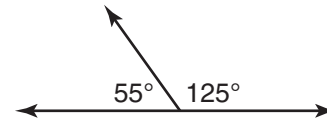**Adjacent angles** are two angles that have a common ray between them.

Supplementary angles are two angles that together can form a straight angle.

The sum of their measures is 180°.

55° 125°

For **1** through **3**, find x.

1.
25°
$x°$

$90° - 25° =$ **65°**

2.
110° $x°$

$180° -$ **110°** $=$ **70°**

3.
$x°$
135°
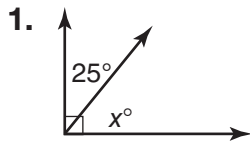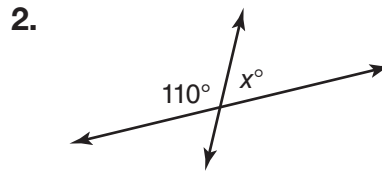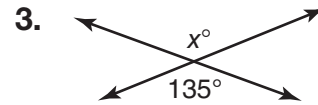
135°

For **4** and **5,** find the measure of an angle that is complementary to an angle with each measure.

4. 15°   **5.** 80°

75°   **10°**

For **6** and **7,** find the measure of an angle that is supplementary to an angle with each measure.

6. 5°   **7.** 100°

175°   **80°**

8. Critical Thinking Which pair of angles are NOT adjacent?

   A w and x

   B x and y

   Ⓒ w and y

   D z and w

w x
z y
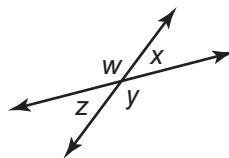
Angle Pairs

For **1** through **3**, find *x*.

1.

21°

x°

69°

2.

x°

122°

122°

3.

x°

108°

72°

For **4** and **5**, find the measure of an angle that is complementary to an angle with each measure.
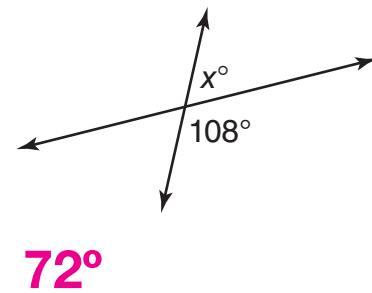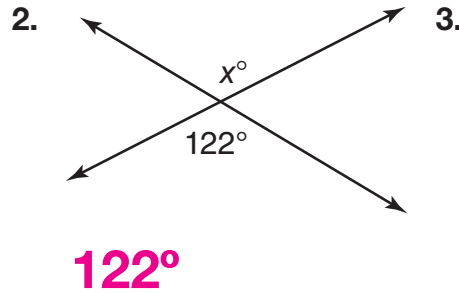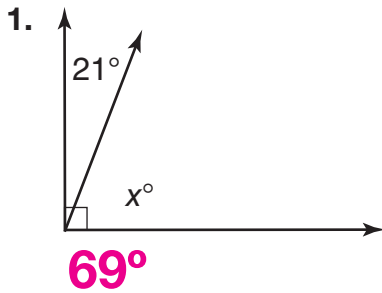
For **6** and **7**, find the measure of an angle that is supplementary to an angle with each measure.

4. 43°

47°

5. 72°

18°

6. 54°

126°

7. 119°

61°

Use the diagram for **8** through **10.**

8. Name two pairs of supplementary angles.
∠CAD and ∠DAE, ∠CAB and ∠BAE.

9. Name two angles adjacent to *DAE.*
∠CAD and ∠BAE.

C

D

A

B

E
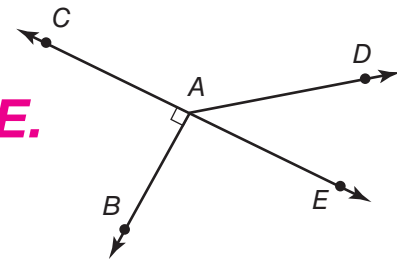
10. Writing to Explain How could you draw an angle complementary to ∠*DAE* without using a protractor? Tell why your method works.

Sample answer: Extend $\overrightarrow{AB}$ to form a line that is perpendicular to $\overleftrightarrow{CE}$. Label this line $\overleftrightarrow{FB}$. ∠FAD is complemetary to ∠DAE.

11. Critical Thinking Which statement is **NOT** true for a pair of intersecting lines?

 A They form two pairs of congruent angles.

 B They form four pairs of complementary angles.

 C They form four pairs of supplementary angles.

 D They form two pairs of vertical angles.

Triangles

Triangles can be classified by their angles or their sides.

Classified by angles

Acute triangle
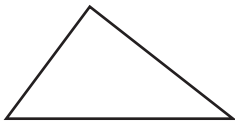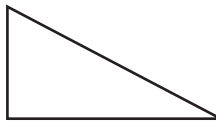All three angles are
acute angles.

Right triangle
One angle is a right
angle.
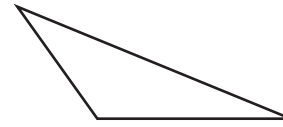
Obtuse triangle
One angle is an
obtuse angle.

Classified by sides
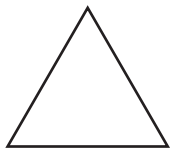
Equilateral triangle
All sides are
congruent.

Isosceles triangle
At least two sides
are congruent.
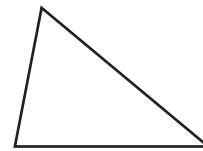
Scalene triangle
No sides are
congruent.

How to find angle measures in a triangle:

Find the measure of angle x.

Remember, when you add up all three
angle measures the sum must be 180°.

$x + 110 + 40 = 180$

$x + 150 = 180$

$x = 30$

Find the missing angle measure. Then classify the triangle by its angles and by its sides.

1.

$x = 48°$; right
and scalene

2.

$x = 60°$; acute
and equilateral

Triangles

Find the missing angle measure. Then classify the triangle by its angles and by its sides.

1.

40°; acute, isosceles

2.
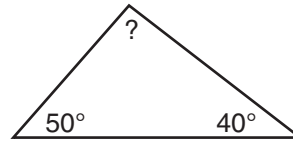
90°; right, scalene

Draw the described triangle.

3. An obtuse scalene triangle

Students should draw an obtuse scalene triangle.

4. A triangle with a 2–inch side between two 50° angles

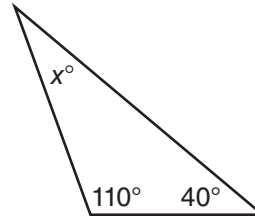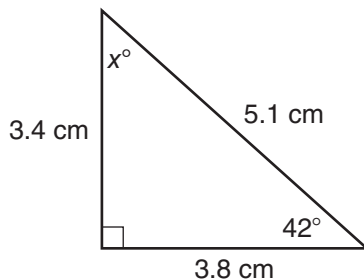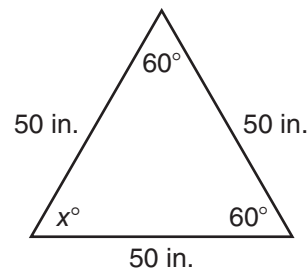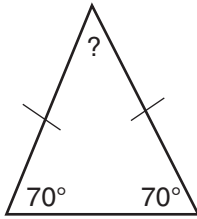**Students should draw a triangle with a 2–inch side between two 50° angles.**

5. Reasoning Can a scalene triangle have two congruent angles? Why or why not? **No, a scalene triangle cannot have two congruent angles. A triangle that has two congruent sides will have two congruent angles, but none of the sides of a scalene triangle are congruent. So a scalene triangle will not have two congruent angles.**

6. A right triangle has a 28° angle. What are the measures of the other angles?

A 28° and 62°

B 28° and 90°

C 62° and 90°

D 62° and 118°

7. Writing to Explain Are all equilateral triangles acute triangles? Explain.

Yes, because all the angles in an equilateral triangle measure 60°.

Quadrilaterals

Classifying quadrilaterals

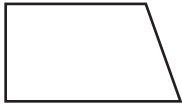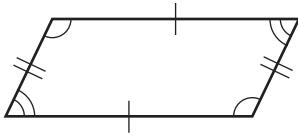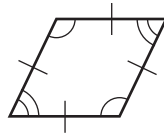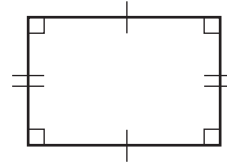| **Trapezoid**	**Parallelogram**	**Rhombus**	**Rectangle**	**Square**
A quadrilateral with only one pair of parallel sides	A quadrilateral with both pairs of opposite sides parallel; Opposite sides and opposite angles are congruent.	A parallelogram with all sides congruent	A parallelogram with four right angles	A rectangle with all sides congruent; A square is also a rhombus.
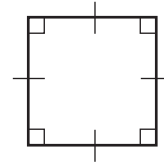
Finding the missing measure of a quadrilateral:

The measures of three angles of a quadrilateral are 115°, 68°, and 45°. Find the measure of the fourth angle.

Remember, the sum of all four angles must be 360°.

$115 + 68 + 45 + x = 360$

$228 + x = 360$

$x = 132$

The measure of the fourth angle is 132°.

Classify each polygon in as many ways as possible.

1.

Quadrilateral, trapezoid

2.
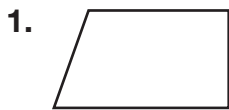
Quadrilateral, parallelogram, rectangle

The measures of three angles of a quadrilateral are given. Find the measure of the fourth angle.

3. 90°, 90°, 90° **90°**

4. 80°, 60°, 120° **100°**

5. 70°, 120°, 120° **50°**

6. 130°, 40°, 50° **140°**

Quadrilaterals

Classify each polygon in as many ways as possible.

1.
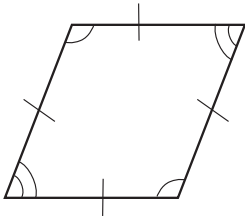
Quadrilateral, parallelogram, rhombus

2.

Quadrilateral, parallelogram, rectangle, rhombus, square

3.

Quadrilateral, trapezoid

The measures of three angles of a quadrilateral are given. Find the measure of the fourth angle and classify each quadrilateral according to its angles.

4. 125°, 55°, 125°
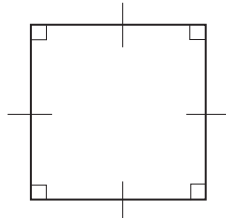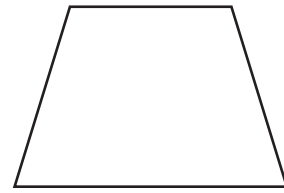
55°, parallelogram or possible rhombus

5. 110°, 100°, 80°
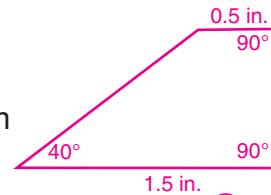
70°, trapezoid

6. 90°, 70°, 150°

50°, quadrilateral only

7. Draw a quadrilateral with one pair of parallel sides. One side is 1.5 in. The other side is 0.5 in. The bottom right and top right angles are 90°. The bottom left angle is 40°. Label the sides and angles.

0.5 in.
90°
40°   90°
1.5 in.

Sample answer:

8. A rhombus has one 65° angle and a 5 cm side. Is this enough information to find the remaining angles and side lengths? Explain.

Sample answer:

Yes, because opposite angles are congruent and all sides are congruent; all side lengths are 5 cm and angles are 65°, 115°, 65°, and 115°.

9. Which pair of angles would be side-by-side in a parallelogram?

A 40°, 40°   **(B)** 40°, 140°   **C** 60°, 110°   **D** 65°, 105°

10. Writing to Explain What characteristics help you classify a quadrilateral as a parallelogram and not a rectangle? Explain.

Sample answer:

A parallelogram has both pairs of opposite sides parallel and congruent, but may not have right angles.

Circles

Radius
Line segment that connects the center to a point on the circle

Arc
Part of a circle connecting two points of the circle

Diameter
Line segment through the center of the circle that connects two points on the circle

Central angle
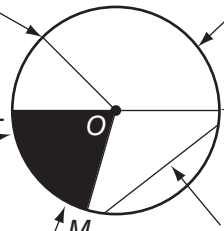Angle whose vertex is the center; ∠LOM is a central angle.

L

O

M

Sector
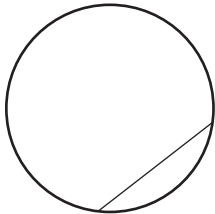Region between two radii and an arc

Chord
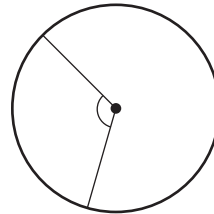Line segment that connects two points on the circle

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Identify the figure or portion of the figure that is drawn in each circle.
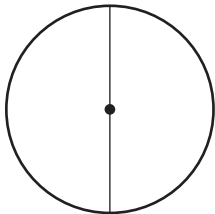
1.

Chord _____

2.

Central angle _____

3.

Diameter _____

4.
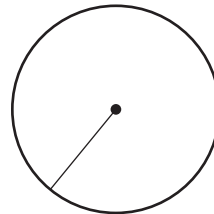
Radius _____

5.
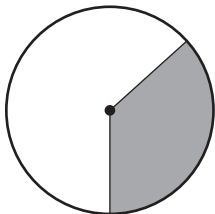
Sector _____

6.
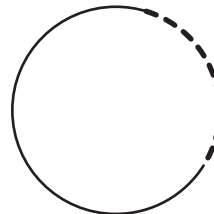
Arc _____

Circles

Identify the figure shown in bold.

1.
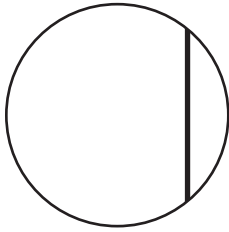
Chord

2.
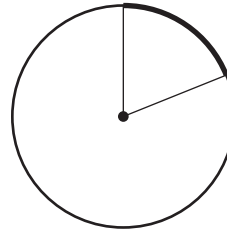
Arc

3.
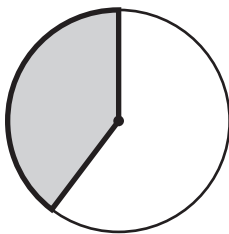
Sector

4.
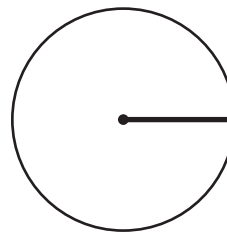
Radius

5. What part of the circle is line segment FG?

Diameter
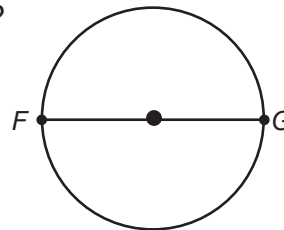
6. How many degrees are in a circle?

   A 90°

   B 120°

   C 180°

   D 360°

7. **Writing to Explain** Explain the relationship between the radius and the diameter of a circle.

The length of the radius is equal to half the length of the diameter.

Transformations and Congruence

Congruent figures have the same size and shape. In congruent shapes, corresponding angles and corresponding sides are congruent. You can use this relationship to find the measures of different angles and different sides. The symbol for congruence is ≅.

The figures at the right are congruent.

∠ABC ≅ ∠QRS, so ∠QRS = 55°
∠BCD ≅ ∠RST, so ∠RST = 125°
$\overline{BC}$ ≅ $\overline{RS}$, so $\overline{RS}$ = 9 cm
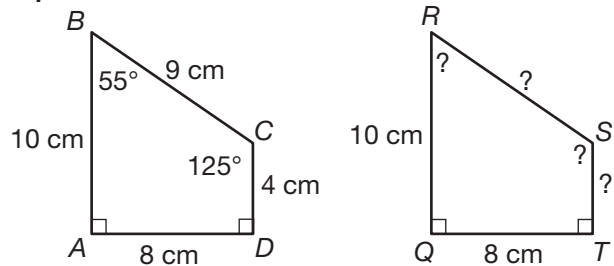$\overline{CD}$ ≅ $\overline{ST}$, so $\overline{ST}$ = 4 cm

A transformation moves a figure to a new position without changing its size or shape.

| A **translation** moves a figure in a straight direction. | A **reflection** gives a figure its mirror image over a line. | A **rotation** moves a figure about a point. | A **glide reflection** is a translation followed by a reflection. |

These figures are congruent. Find the angle and side measures.

1.

$\overline{MP}$ = **3 in.** $\overline{MN}$ = **6 in.**

2.
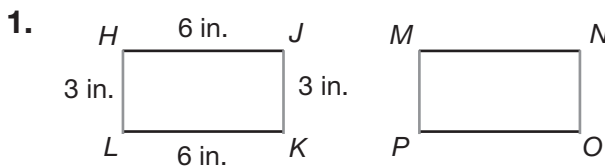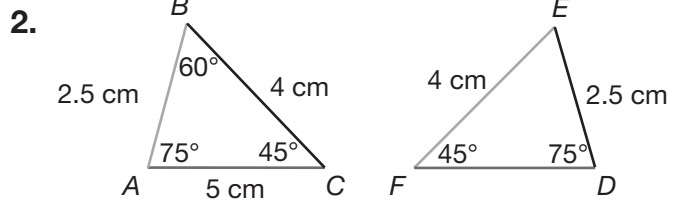
$\overline{FD}$ = **5 cm**   m∠FED = **60°**

Tell whether the figures in each pair are related by a translation, a reflection, a glide reflection, or a rotation.
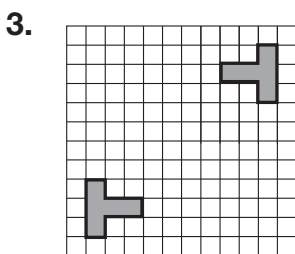
3. **Rotation or Glide reflection**
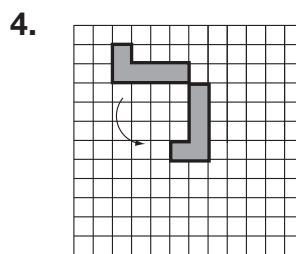
4. **Rotation**

5. Writing to Explain Describe the relationship between the two triangles in Item 2.

Sample answer: The triangles show a reflection.

Transformations and Congruence

1. These parallelograms are congruent. Find $\overline{CD}$, $\overline{GH}$, and $m\angle D$.

$\overline{CD} = $ **$1\frac{1}{2}$ ft** $\overline{GH} = $ **2 ft**

$m\angle D$ **120°**

F $1\frac{1}{2}$ ft
120° *G*
E
$1\frac{1}{2}$ ft *H*

B   *C*
60°
A   2 ft   *D*

Tell whether the figures in each pair are related by a translation, a reflection, a glide reflection, or a rotation. If the relationship is a rotation, describe it.

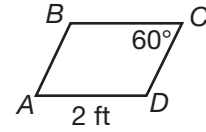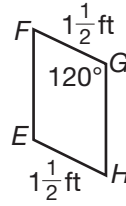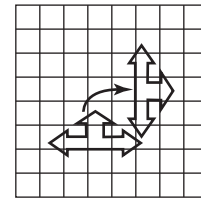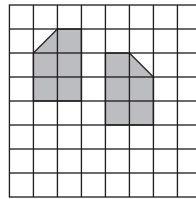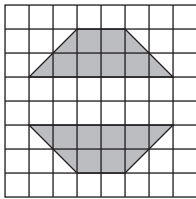**2.** **See below.**   **3.** **Glide reflection**   **4.** **Rotation, 90° clockwise**

5. Use the grid. Draw a semi-circle to the left of the *y*-axis. Then show the semi-circle reflected across the *y*-axis.

Sample answer shown.
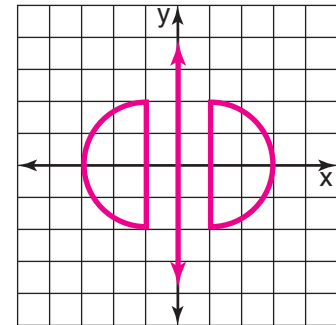
6. Cole drew two congruent polygons. Which is true about *all* congruent figures?

(A) Corresponding angles are congruent.

B Corresponding angles are complementary.

C Corresponding angles are supplementary.

D There are no corresponding angles.

7. Writing to Explain Draw a figure. Use different transformations of your figure to make a pattern. Show three repetitions. Then explain which transformations are used in your pattern.

Sample answer: The pattern is translation, reflection.

2. Reflection, glide reflection, or 180° rotation

Symmetry

A figure has **reflection symmetry** if it can be reflected onto itself. The line of reflection is called the **line of symmetry**. Some figures have more than one line of symmetry.

One line of symmetry
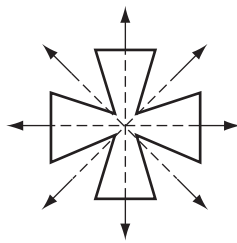
Four lines of symmetry
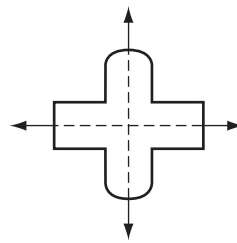
Two lines of symmetry

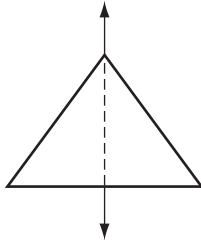A figure has **rotational symmetry** when it rotates onto itself in less than one full turn.
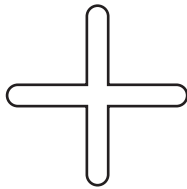
90° ($\frac{1}{4}$ turn) rotational symmetry
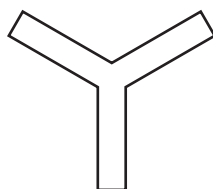
120° ($\frac{1}{3}$ turn) rotational symmetry
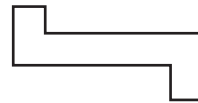
180° ($\frac{1}{2}$ turn) rotational symmetry
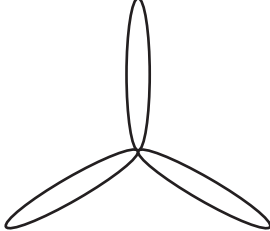
Tell if each figure has reflection symmetry, rotational symmetry, or both. If it has reflection symmetry, how many lines of symmetry are there? If it has rotational symmetry, what is the smallest turn that will rotate the figure onto itself?

1. **Reflection symmetry; 1 line**

2. **Both; 4 lines; $\frac{1}{4}$ turn**

3. **Rotational symmetry; $\frac{1}{2}$ turn**

4. **Both; 3 lines; $\frac{1}{3}$ turn**

Symmetry

Tell if each figure has reflection symmetry, rotational symmetry, or both.
If it has reflection symmetry, how many lines of symmetry are there? If
it has rotational symmetry, what is the smallest turn that will rotate the
figure onto itself?

1. **Both,
2 lines, 180°**
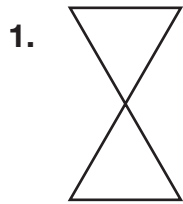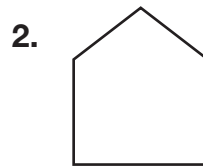
2. **Reflection,
1 line**
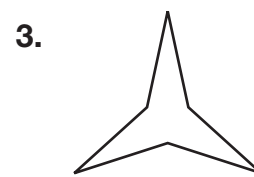
3. **Both,
3 lines, 120°**

4. Reasoning Describe the symmetry of an equilateral triangle.

**3 lines of reflection symmetry; 120°
rotational symmetry**

5. 808 is an example of a number with reflection symmetry.
Write another number that has reflection symmetry.

Sample answer: l8l

6. Which does the figure have?

(**A**) Rotational symmetry

B Reflection symmetry

C Neither

D Both
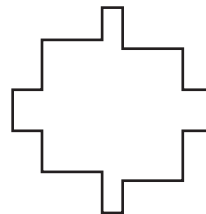
7. Writing to Explain Draw a figure with reflection symmetry,
and draw the line of symmetry.

Sample answer:
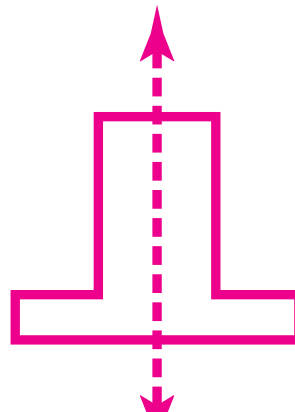
Problem Solving: Make a Table and Look for a Pattern

How could you explain the pattern shown by the blocks? How many blocks will there be in the 10th element of the pattern?

Make a table to show the number of blocks in each element.

Element number	1	2	3	4	5	6	7	8	9	10
Number of blocks	1	3	5	7	**9**	**11**	**13**	**15**	**17**	**19**

The pattern: Each element has 2 more blocks than the element before it.

Write a mathematical expression you can use to find the number of blocks in any element.

$x(2) - 1$, where x is the element number

Evaluate for the 10th element:

$10(2) - 1 = 20 - 1 = 19$

The 10th element will have 19 blocks.

1 2 3 4
Element

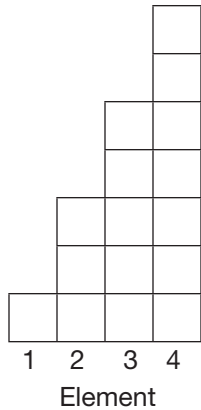**1.** What is the pattern in this table? Write this as an expression.

Row *a*	1	2	3	4	5
Row *b*	4	7	10	13	16

The *a* numbers are just counting numbers in order. You can find any *b* element of the pattern using the expression $3a + 1 = b$.

2. The table shows the sum of the interior angles of several polygons. What is the sum of the interior angles in a regular polygon with 14 sides?

Number of sides	3	4	5	6	7	8
Sum of angles	180°	360°	540°	**720°**	**900°**	**1,080°**

The sum of the interior angles of a polygon with 14 sides is 2,160°.

3. A quarry charges $56.00 per ton of gravel. A discount of $3.00 is given for buying 2 tons, $6.00 for buying 3 tons, and so on. What would the discount be for buying 12 tons of gravel?

The discount for buying of 12 tons of gravel is $33.00.

4. The first square in a pattern is 1 cm on a side. Each square after that adds 1 cm to each side. What is the area of the 7th square?

The area of the seventh square is 49 cm².

Problem Solving: Make a Table and Look for a Pattern

1. Find the next three numbers in each row. Write a formula to find any number in row B.

$B = 3A - 4$

A	2	4	6	**8**	**10**	**12**
B	2	8	14	**20**	**26**	**32**

2. A company offers a 2% discount if you buy 1–5 of their products. If you buy 6–10 of their products, you earn a 3.5% discount. Buying 11–15 products will earn you a 5% discount. If the pattern continues, what discount would be offered for buying 33 products?

11% discount

3. Explain the pattern. Draw the next eleven shapes.

○□○△○□□○△△○□□□○△△△○□□□□○△
△△△○□□□□□○△

The general pattern is circle-square-circle-triangle, but the square and triangle are increased by 1 in each step of the pattern.

4. In a contest, the first place team gets $\frac{1}{2}$ of the million-dollar prize. The second place team gets $\frac{1}{2}$ of the remaining money. Each team after that gets $\frac{1}{2}$ of the remaining money. How much will the sixth place team get?

$15,625

5. An advertising sign lights up for 5 seconds then goes out for 2 seconds. For how many seconds will the sign be off in the first minute after the sign is turned on?

A 46 seconds **B** 30 seconds **C** 16 seconds **D** 2 seconds

6. Writing to Explain Explain your thinking as you find how many triangles would be in the 8th row of the pattern
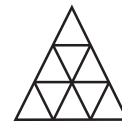
Sample answer: For the eighth row, you multiply 8 × 2 and subtract 1 to get 15 triangles.