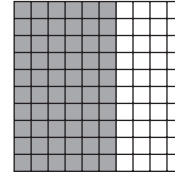# Understanding Percent

A percent is a ratio that compares a part to a whole.
The second term in the ratio is always 100.
The whole is 100%.
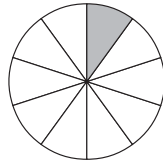The grid has 60 of 100 squares shaded.

$\frac{60}{100} = 60\%$

So, 60% of the grid is shaded.

When the second term of a ratio is not 100, you can write an equivalent ratio with a denominator of 100 or use a proportion to find the percent shown by the part.
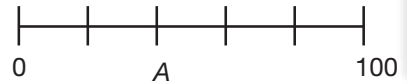
**The circle has 1 of 10 equal-size parts shaded.**

$\frac{1}{10} = \frac{10}{100} = 10\%$ or $\frac{1}{10} = \frac{x}{100}$
$10x = 100$
$x = 10$

So, 10% of the circle is shaded.

The line segment represents 100%. What percent is shown by Point $A$?

$\frac{2}{5} = \frac{40}{100} = 40\%$ or $\frac{2}{5} = \frac{x}{100}$
$5x = 200$
$x = 40$

So, 40% of the line segment is shaded.

---

Write the percent of each figure that is shaded.

**1.** **94%**

**2.** **3%**

**3.** **40%**

**4.** **50%**

**5.** **75%**
0      $\frac{6}{8}$    100

**6.** **19%**
0   19      100

**7. Number Sense** Jana divided a sheet of paper into 5 equal sections and colored 2 of the sections red. What percent of the paper did she color? **40%**

**8. Writing to Explain** Shade each model to show 100%. Explain how you knew how many parts to shade. **Since the whole is 100%, I knew I should shade all of each model.**
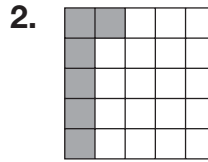
0                 100
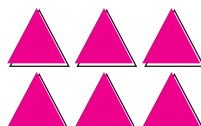
# Understanding Percent

Write the percent of each figure that is shaded.

**1.**



**81%**

**2.**



**24%**

**3.**



**60%**

**4.**
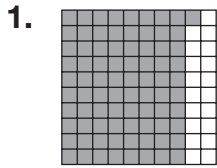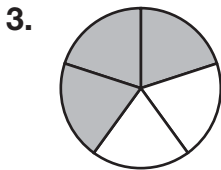


**90%**

**5. Number Sense** What percent of line segment *AB* is equal to 50% of line segment *CD*?
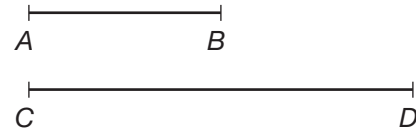
**100%**



**6.** The line segment below shows 100%. Show 25%, 50%, and 75% of the segment.

**25%     50%     75%**



**7.** Which of the following figures is 60% shaded?

**Ⓐ**   **B**   **C**   **D**
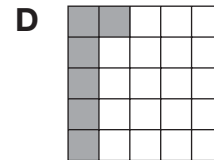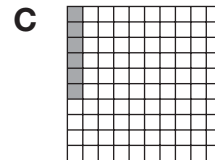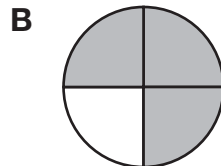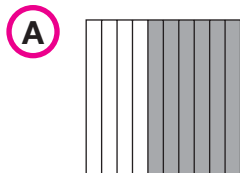


**8. Writing to Explain** You are thirsty, so a friend has offered to give you 50% of his water. What information must you have in order to find out how much water your friend will give you?

**Sample answer: The total amount of water your friend has**

# Fractions, Decimals, and Percents

Fractions, decimals, and percents all name parts of a whole.
Percent means per hundred, so 15% means 15 parts per hundred.
The grid to the right has 72 out of 100 squares shaded. The shaded part
can be represented with a fraction, $\frac{72}{100}$ ($\frac{18}{25}$ in simplest form), by a decimal,
0.72, and by a percent, 72%.

**Write 36% as a fraction in simplest form and as a decimal.**

$36\% = \frac{36}{100} = 0.36$

Simplify the fraction:

$\frac{36}{100} = \frac{36 \div 4}{100 \div 4} = \frac{9}{25}$

So, $36\% = \frac{9}{25} = 0.36$.

**Write 0.47 as a fraction in simplest form and as a percent.**

$0.47 = \frac{47}{100} = 47\%$

**Write $\frac{3}{4}$ as a decimal and as a percent.**

You can use a proportion or divide to help you.

Use a proportion:

$\frac{3}{4} = \frac{n}{100}$

$4n = 300$

$n = 75$

Use division:

$$4\overline{)3.00}$$
$$\begin{array}{r} 0.75 \\ \underline{2\ 8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$
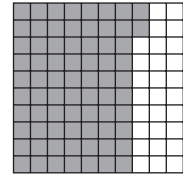
So, $\frac{3}{4} = \frac{75}{100} = 0.75 = 75\%$.

Write each number in two other ways. Write fractions in simplest form.

**1.** $\frac{2}{100}$  **0.02** ; **2%**

**2.** $\frac{71}{100}$  **0.71** ; **71%**

**3.** $\frac{9}{10}$  **0.9** ; **90%**

**4.** 17%  **0.17** ; $\frac{17}{100}$

**5.** 48%  **0.48** ; $\frac{12}{25}$

**6.** 60%  **0.6** ; $\frac{3}{5}$

**7.** 0.04  $\frac{1}{25}$ ; **4%**

**8.** 0.22  $\frac{11}{50}$ ; **22%**

**9. Writing to Explain** Jamal said that he could write a percent as a decimal by moving the decimal point two places to the left and deleting the percent sign. Is he correct? How do you know?

**Yes, I looked at the exercises and his conjecture worked for those numbers.**

**10. Number Sense** Two stores sell their goods at the manufacturers' suggested retail prices, so their prices are the same. Which store has the greatest markdown from their original prices?

| GOODS 2 GO | BUY AND BYE |
|---|---|
| $\frac{1}{4}$ off original prices! | 30% off original prices! |

**Buy and Bye; $\frac{1}{4}$ = 25%, 30 > 25.**

# Fractions, Decimals, and Percents

Describe the shaded portion of each as a fraction, decimal, and percent.

**1.**



$\frac{43}{100}$, **0.43, 43%**

**2.**
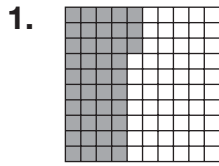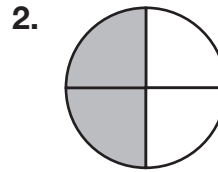


$\frac{2}{4}$, **0.5, 50%**

Write each in two other ways.

**3.** 64%

**0.64, $\frac{64}{100}$**

**4.** 0.09

**9%, $\frac{9}{100}$**

**5.** $\frac{12}{50}$

**24%, 0.24**

**6.** 37%

$\frac{37}{100}$, **0.37**

**7.** $\frac{4}{250}$

**0.016, 1.6%**

**8.** 0.023

$\frac{2.3}{100}$, **2.3%**

The table at the right shows the number of states in the United States at different times in history. There are currently 50 states in the United States. Use the information to answer the questions.

| Year | States |
|------|--------|
| 1792 | 15 |
| 1817 | 20 |
| 1836 | 25 |
| 1848 | 30 |
| 1863 | 35 |
| 1889 | 40 |
| 1896 | 45 |
| 1959 | 50 |

**9.** In what year were there 0.5 as many states as today?

**1836**

**10.** What percent of the current number of states had joined the United States by the year 1863?

**70%**

**11.** In what year were there about $\frac{2}{3}$ as many states as in 1896? **1848**

**12.** Which of the following is equivalent to 98%?

**A** 0.49     **B** $\frac{100}{98}$     **(C)** 0.98     **D** $\frac{49}{100}$

**13. Writing to Explain** Explain how you would write $\frac{5}{6}$ as a percent.

**Sample answer:**
**I would use a proportion: $\frac{5}{6} = \frac{X}{100}$.**

# Percents Greater Than 100 and Less Than 1

All percents can be written as fractions in simplest form and as decimals. Percents greater than 100% represent amounts greater than one whole and can be written as improper fractions and as decimals greater than 1. Percents less than 1% represent amounts less than $\frac{1}{100}$ of the whole.

**Write 275% as a fraction in simplest form and as a decimal.**

Since percent is parts per hundred, write the percent as a fraction with a denominator of 100.

$\frac{275}{100}$

Simplify the fraction.

$\frac{275}{100} = \frac{275 \div 25}{100 \div 25} = \frac{11}{4} = 2\frac{3}{4}$

To write the number as a decimal, divide the numerator by the denominator.
So, $275\% = 2\frac{3}{4} = 2.75$

$275 \div 100 = 2.75$

**Write $\frac{1}{5}$% as a fraction in simplest form and as a decimal.**

Write the fraction in the percent as a decimal.

$\frac{1}{5}\% = 0.2\%$

Write the percent as a fraction with a denominator of 100.

$\frac{0.2}{100}$

Write the numerator as a whole number.

$\frac{0.2}{100} = \frac{0.2 \times 10}{100 \times 10} = \frac{2}{1,000}$

Simplify the fraction.

$\frac{2}{1,000} = \frac{2 \div 2}{1,000 \div 2} = \frac{1}{500}$

Divide the fraction to write the number as a decimal.

$\frac{1}{500} = 0.002$

So, $\frac{1}{5}\% = \frac{1}{500} = 0.002$.

Write each percent as a fraction and as a decimal. Write fractions in simplest form.

1. 137%  $\frac{137}{100}$ ; **1.37**

2. 115%  $\frac{23}{20}$ ; **1.15**

3. 222%  $\frac{111}{50}$ ; **2.22**

4. 500%  $\frac{5}{1}$ ; **5.0**

5. 182%  $\frac{91}{50}$ ; **1.82**

6. 450%  $\frac{9}{2}$ ; **4.5**

7. 0.4%  $= \dfrac{\boxed{0.4}}{100} = \dfrac{\boxed{0.4} \times 10}{100 \times 10} = \dfrac{\boxed{4}}{\boxed{1,000}}$ ; Simplify: $\dfrac{\boxed{1}}{\boxed{250}}$ ; Decimal: **0.004**

8. $\frac{3}{4}$%  $= \dfrac{0.75}{100} = \dfrac{\boxed{0.75} \times 100}{100 \times 100} = \dfrac{\boxed{75}}{\boxed{10,000}}$ ; Simplify: $\dfrac{\boxed{3}}{\boxed{400}}$ ; Decimal: **0.0075**
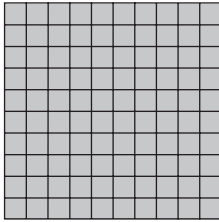
9. **Writing to Explain** Caryn and Alfonso bought school supplies. Caryn spent 130% of the amount Alfonso spent. She said that she spent 1.3 times the amount that Alfonso spent. Is Caryn correct? Explain.

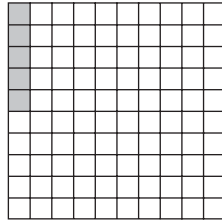   **Yes, 130% = 1.3, so Caryn is correct.**

# Percents Greater Than 100 and Less Than 1

Write a fraction in simplest form, a decimal, and a percent to name each shaded part.

**1.**

$\frac{21}{20}$, **1.05, 105%**

**2.**
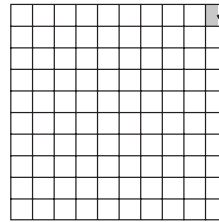
$\frac{3}{500}$, **0.006, 0.6%**

Write each percent as a fraction and as a decimal. Write fractions in simplest form.

**3.** 188% $\frac{47}{25}$ ; **1.88**

**4.** 145% $\frac{29}{20}$ ; **1.45**

**5.** 261% $\frac{261}{100}$ ; **2.61**

**6.** 350% $\frac{7}{2}$ ; **3.5**

**7.** 275% $\frac{11}{4}$ ; **2.75**

**8.** 420% $\frac{21}{5}$ ; **4.2**

**9.** 400% $\frac{4}{1}$ ; **4.0**

**10.** $\frac{1}{5}$% $\frac{1}{500}$ ; **0.002**

**11.** 0.7% $\frac{7}{1,000}$ ; **0.007**

**12.** $\frac{1}{3}$% $\frac{1}{300}$ ; **0.00$\overline{3}$**

**13.** The land area of Yosemite National Park is 3079 km². This is about 189% of the land area of Sequoia National Park. Write 189% as a fraction in simplest form and as a decimal.

**A** $\frac{100}{189}$, 0.53 (rounded)

**C** $\frac{189}{100}$, 18.9

**B** $\frac{189}{100}$, 1.89

**D** $\frac{3079}{189}$, 16.29

**14. Writing to Explain** Nathan wanted to save $400 for a new bicycle. He saved 110% of his goal amount. Write 110% as a fraction in simplest form and as a decimal. Has he saved enough money to buy the bicycle? Explain how you know.

$\frac{11}{10}$, **1.1; He has saved more than enough money because 100%, the whole, is the goal amount. A percent over 100% is greater than the whole. Since Nathan has saved 110%, he has saved $440, which is more than the goal.**

# Estimating Percent

**Estimate 8% of 300,000.**

Round the percent.
  8% ≈ 10%

Think of the equivalent decimal.
  10% = 0.1

Multiply.
  $0.1 \times 300{,}000 = 30{,}000$

**Estimate 27% of 297.**

Round both numbers.
  27% ≈ 30%    297 ≈ 300

Think of an equivalent decimal.
  30% = 0.3

Multiply.
  $0.3 \times 300 = 90$

To multiply by 0.1, move the decimal point one place to the left.
  $0.1 \times 50 = 5$        $0.1 \times 4700 = 470$        $0.1 \times 3{,}659 = 365.9$

To multiply by a multiple of 0.1, such as 0.3, break apart the number.
  $0.3 = 0.1 \times 3$

Multiply one step at a time.
  $0.1 \times 300 = 30$        $30 \times 3 = 90$

Round each percent, then write the equivalent decimal.

**1.** 41%  **40%, 0.4**    **2.** 88%  **90%, 0.9**    **3.** 76%  **80%, 0.8**

**4.** 22%  **20%, 0.2**    **5.** 37%  **40%, 0.4**    **6.** 59%  **60%, 0.6**

Break apart each decimal so the numbers are easier to multiply.

**7.** 0.4  **0.1 × 4**    **8.** 0.9  **0.1 × 9**    **9.** 0.6  **0.1 × 6**

Estimate each percent.

**10.** 9% of 20  **2**        **11.** 21% of 31  **6**        **12.** 31% of 37  **12**

**13.** 38% of 49  **20**      **14.** 49% of 101  **50**      **15.** 61% of 19  **12**

**16.** 59% of 304  **180**    **17.** 70% of 471  **350**     **18.** 84% of 149  **120**

**19.** **Number Sense** What is another way to estimate 51% of 42?
**51% is close to 50%, which equals $\frac{1}{2}$. 42 ÷ 2 = 21.**

**20.** **Reasoning** If 10% of a number is 100, what is 15% of that number? Explain how you determined your answer. **If 10% of a number is 100, then 5% of the same number is 50.  10% + 5% = 15%, so 15% of the number is 100 + 50 = 150.**

# Estimating Percent

Estimate.   **Sample answers are given for 1-19.**

**1.** 35% of 102 **35**

**2.** 42% of 307 **120**

**3.** 79% of 13 **8**

**4.** 84% of 897 **720**

**5.** 13% of 97 **13**

**6.** 28% of 95 **30**

**7.** 61% of 211 **120**

**8.** 19% of 489 **100**

**9.** 48% of 641 **320**

**10.** 21% of 411 **80**

**11.** 77% of 164 **128**

**12.** 51% of 894 **450**

**13.** 39% of 306 **120**

**14.** 62% of 522 **300**

**15.** 48% of 341 **170**

**16. Number Sense** Which would you need to estimate to find an answer, 45% of 200 or 46% of 97?

**46% of 97**

**17.** The school store sold 48 items on Monday. Of those items, 60% were pens. About how many pens were sold on Monday?

**About 30 pens**

**18.** The school cafeteria workers cooked 52 lb of pasta on Thursday. Of that, 90% was sold on Thursday, and 10% was stored in the refrigerator. About how much pasta was stored in the refrigerator?

**About 5 lb**

**19.** On a rainy day, 76% of the students in the school brought umbrellas. There are 600 students in the school. About how many students brought umbrellas?

**About 450 students**

**20.** Which of the following is the best estimate for 68% of 251?

**A** 150

**B** 175

**C** 204

**D** 210

**21. Writing to Explain** Explain how you would estimate 79% of 389.

**Sample answer: To find an estimate, round 79% to 80% and 389 to 400. So, 80% of 400 is 320.**

# Finding the Percent of a Number

**Find 77% of 240.**

First estimate.
$$77\% \approx 75\% = \frac{3}{4}$$
$$\frac{3}{4} \times 240 = 180$$

Use a decimal.
Change the percent to a decimal.
$$77\% = 0.77$$

Multiply.
$$0.77 \times 240 = 184.8$$

The answer 184.8 is close to the estimate 180.

Use a proportion.

Write the percent as a fraction.
$$77\% = \frac{77}{100}$$

Write the proportion and solve.
$$\frac{x}{240} = \frac{77}{100}$$
$$100x = 18{,}480$$
$$\frac{100x}{100} = \frac{18{,}480}{100}$$
$$x = 184.8$$

Find the percent of each number.

**1.** 25% of 24 **6**

**2.** 50% of 72 **36**

**3.** 72% of 88 **63.36**

**4.** 18% of 97 **17.46**

**5.** 66% of 843 **556.38**

**6.** 46% of 388 **178.48**

**7.** 89% of 111 **98.79**

**8.** 0.7% of 392 **2.744**

**9.** 110% of 640 **704**

**10. Geometry** Ava's aquarium is 10 in. tall, 15 in. long, and 8 in. wide. The aquarium is 95% filled with water. How many cubic inches of water are in the aquarium?

**1,140 in.$^3$**

**11.** DeWayne used his music club membership card to get 15% off the cost of a CD. If the regular price of the CD was $15.95, how much did DeWayne pay?

**$13.56**

**12.** Marla bought a dress priced at $89.99. She used a 20% off coupon. How much did she pay for the dress?

**$71.99**

**13. Writing to Explain** Tell how you could use a proportion to find 125% of 500. Why is the solution greater than the original number?

**I would use a proportion: $\frac{125}{100} = \frac{x}{500}$. Then, I would think about what number times 100 = 500 and then multiply 125 by that number, 5. The solution is greater than the original number because the percent is greater than 100.**

# Finding the Percent of a Number

Find the percent of each number.

**1.** 42% of 800    **336**    **2.** 5.6% of 425    **23.8**    **3.** 85% of 15    **12.75**

**4.** $33\frac{1}{3}$% of 678    **226**    **5.** 12% of 65    **7.8**    **6.** 58% of 324    **187.92**

**7.** 98% of 422    **413.56**    **8.** 32% of 813.5    **260.32**    **9.** 78% of 219    **170.82**

**10.** 13% of 104    **13.52**    **11.** 24% of 529    **126.96**    **12.** 4.5% of 82    **3.69**

**13.** 64% of 912    **583.68**    **14.** 128% of 256    **327.68**    **15.** 63% of 1,368    **861.84**

**16.** About 42% of the flag of the United States is red. On a flag that is 9 feet tall and 15 feet wide, how many square feet are red?

     **56.7 ft²**

**17. Estimation** Estimate 68% of 32, then find the actual answer. Which is greater?

**Estimate: 24; Actual: 21.76; Estimate is greater.**

For **18** and **19**, round your answer to the nearest whole number.

**18.** An adult has 206 bones. Of those, approximately 2.9% are found in the inner ear. About how many bones in the human body are found in the inner ear?

     **6 bones**

**19.** Approximately 12.6% of the bones are vertebrae in the human back. About how many bones in the human body are vertebrae?

     **26 bones**

**20.** 45 is 12% of which number?

   **A** 540      **B** 450      **C** 375      **D** 5.4

**21. Writing to Explain** Without calculating, tell which is greater, 52% of 3,400 or 98% of 1,500. Explain.

**52% of 3,400 is greater. About half of 3,400 is greater than about all of 1,500.**

# Applying Percents:
# Finding the Whole

You can draw a number line model to help you solve this
problem:

> Darlene spent 10% of her allowance and saved the rest.
> The amount she spent was 50 cents. How much is
> Darlene's allowance?

In the problem, 50 cents is the part and 10% is the percent.
You need to find Darlene's allowance, *a.*
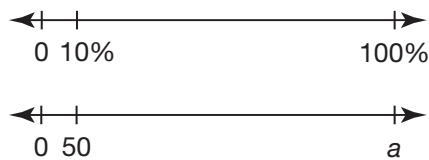


The model shows 10% as the percent, 50 cents as the part, and *a,*
the whole you are trying to find.

A proportion can also help you find the whole.

$\frac{10}{100} = \frac{50}{a}$

$a = 500$

500 cents = $5.00

Darlene's allowance is $5.00.

> Think: 10 times what number equals 50?
> Since 10 × 5 = 50, then
> multiply 100 × 5 to get 500.

For **1** through **3**, draw a number line model to help you solve the
problem. **Check students' drawings.**

1.  Li rode her bike 25% of the way to school. She rode 5 blocks. How
    many blocks does Li live from school?

    **20 blocks**

2.  Bob brought 40% of the collected canned goods to the food pantry. If
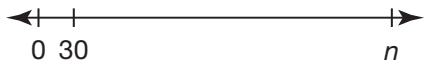    Bob brought 160 cans to the pantry, how many cans were collected?

    **400 cans**

3.  Sid memorized 60% of his lines for the class play. He memorized
    60 lines. How many lines long was Sid's part in the play?

    **100 lines**
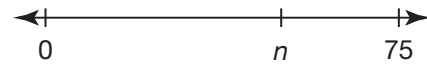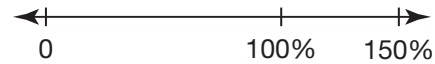
# Applying Percents: Finding the Whole

In **1** through **4**, use the number lines and write a proportion to solve.
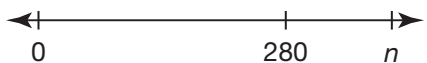
**1.** 10% of what number is 30?

```
◄─┼─┼────────────────┼─►
  0 10%              100%
```
```
◄─┼─┼────────────────┼─►
  0 30               n
```
$\frac{10}{100} = \frac{30}{n}; n = 300$

**2.** 150% of what number is 75?

```
◄─┼────────────┼──────┼─►
  0            100%   150%
```
```
◄─┼────────────┼──────┼─►
  0            n      75
```
$\frac{150}{100} = \frac{75}{n}; n = 50$

**3.** 70% of what number is 280?
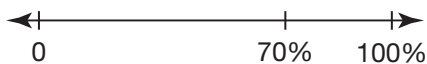
```
◄─┼────────────┼──────┼─►
  0            70%    100%
```
```
◄─┼────────────┼──────┼─►
  0            280    n
```
$\frac{70}{100} = \frac{280}{n}; n = 400$

**4.** 56% of what number is 84?
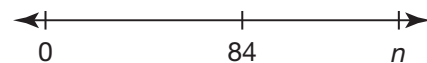
```
◄─┼────────────┼──────┼─►
  0            56%    100%
```
```
◄─┼────────────┼──────┼─►
  0            84     n
```
$\frac{56}{100} = \frac{84}{n}; n = 150$

For **5** through **8**, find each whole.

**5.** 60% of what number is 12?

**20**

**6.** 100% of what number is 61?

**61**

**7.** 40% of what number is 5?
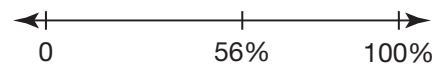
**12.5**

**8.** 14% of what number is 7?

**50**

**9.** Only 5% of the total attendees for a concert have arrived.
If 105 people have arrived, how many attendees are expected?

**(A)** 2,100          **B** 2,025          **C** 1,950          **D** 1,800

**10. Writing to Explain** You want to solve 11% of what number
is 22. Explain how you can do this using a proportion

**Write the first ratio as $\frac{11}{100}$ and set it equal to $\frac{22}{n}$.**
**Since 11 × 2 = 22 you can multiply 100 × 2 to**
**get 200. Check: 11% of 200 = 22**

# Problem Solving: Reasonableness

After solving a problem, look back and check that your answer is reasonable and that you answered the correct question.

Terrell bought a skateboard on sale for 20% off the original price. He also had a coupon for 10% off. The original price was $80. How much did Terrell pay for the skateboard before tax?

*Answer: Terrell paid $24 for the skateboard.*

**Is my answer reasonable?**

Since the discount is about 30% off, Terrell will pay about 70% of the original cost of the skateboard.

70% of $80 is $56.

The answer is not reasonable because the price of the skateboard should be about 70% of the original price, or $56.

**Did I answer the correct question?**

Yes. The question asks for the sale price of the skateboard.

**Ask yourself:**

Did I use the correct operation(s) to solve the problem?

Is all of my work correct?

Is the actual answer close to my estimate?

**Ask yourself:**

What am I asked to find?

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Look back and check. Tell if the answer given is reasonable. Explain why or why not.

**1.** Marita bought some toys for her cat at the pet store. The pet store is having a storewide discount of 15% on all pet toys. How much will Marita pay for the toys if the total price before the discount is $42?
   *Answer: The discount price is $35.70.*
   **Sample answer: The answer is reasonable. 10% of $40 is $4, and 5% of $40 is $2. $40 − $6 is $34 and $34 is close to $35.70.**

**2.** Frankie paid a total of $53.50 for some fish for his aquarium. The price includes a coupon for 7% off. What was the cost of the fish?
   *Answer: The fish cost $50.00.*
   **Sample answer: The answer is not reasonable since the price of the fish before the discount should be more than $53.50.**

# Problem Solving: Reasonableness

Look back and check. Tell if the answer given is reasonable.
Explain why or why not.

1. A shipment of 200 games is 20% video games, 50% board games, and 30% puzzles. How many board games are chess if 25% of the board games are chess?
*Answer: The number of chess games is 50.*
**Sample answer: The answer is not reasonable. 50% of 200 is 100 and 25% of 100 is 25. Chess is 25% of half of the games, not 25% of all of the games.**

2. A DVD player costs $199. How much will it cost if it is 15% off?
*Answer: The cost of the DVD player will be $169.15.*
**Sample answer: The answer is reasonable. 10% of $200 is $20, so 5% of $200 is $10. $200 − $30 is $170 and $170 is close to $169.15.**

3. **Write a Problem** An ad in the newspaper is offering 25% off ski lift tickets at Big Bear. The original tickets cost $60. Write a problem using the information from the ad. Then give an answer for someone to look back and check for reasonableness.
**Sample answer: What is the cost of ski lift tickets if the original price is $60 and a newspaper ad is offering 25% off? Answer: The cost is $45.**

4. Students at Warm Springs Middle School are going on a field trip to Orange County. If 60% of the 120 students signed up for the field trip are girls, and 25% of the girls are in sixth grade, how many sixth grade girls are going on the field trip?

   **(A)** 18          **B** 25          **C** 43          **D** 102

5. **Writing to Explain** Bailey paid $42 for a backpack that was 40% off the original price. Is $56 a reasonable price for the original cost of the backpack? Explain.
**Sample answer: No, since you pay 60% if the price is 40% off and 60% of $70 is $42, the original cost is $70.**